ity; $Sh_1 = f_1d/U_0$, dimensionless cylinder vibrations frequency, the kinematic Strouhal number; $Sh = fd/U_0$, Strouhal number of vortex shedding; $Re = dU_0/\nu$, Reynold number; $\tau = tU_0/d$, dimensionless time; $\bar{y} = y/d$; $\bar{y}_0 = y_0/d$; $\bar{l} = l/d$; $\bar{u}/U_0$; t_1, period of cylinder vibrations, and $T = t_1U_0/d$, dimensionless period of vibrations.

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DISTRIBUTION OF VELOCITY PULSATIONS IN A CHANNEL WITH MIXING OF OPPOSITELY SWIRLED STREAMS

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The influence of swirling and of the degree of concurrent flow on the magnitude and distribution of turbulent pulsations in the mixing of oppositely swirled streams is investigated. The correspondence between the pulsation characteristics of the mixing layers of swirled flows and concurrent jets is established.

Experimental data [1, 2] show that there is a generality in the mechanisms of mixing of oppositely swirled streams and concurrent jets, which is manifested most clearly in flows with significant transverse shear. For example, it was shown in [2] that the relative velocity profiles in a mixing layer of oppositely swirled streams with a degree of concurrent flow $m = 1$ are self-similar with respect to the channel length and they coincide in shape with the corresponding profiles in the mixing layers of concurrent unswirled jets. The analogy in the laws of expansion of these mixing layers is established.

The analysis carried out in [2] was based on hypotheses that the rate of growth of the mixing layer is proportional to the magnitude of the transverse pulsation velocity, which in turn is proportional to the transverse gradient of the averaged velocity [3]. Such an approach is evidently unsuitable for the analysis of flow of a more complicated form, and one must turn to the investigation of the pulsation characteristics of the flow, which characterize the processes of turbulent exchange directly, without the resort to additional hypotheses. The present article is devoted to the consideration of this question. The investigation carried out in it is a continuation of [2] and is directed toward the search for regularities in the development of the pulsation characteristics of flow arising during the mixing of coaxial, oppositely swirled streams in a channel and their comparison with the corresponding characteristics of unswirled flows with transverse shear.

The experiments were carried out on an installation, the working part of which consisted of an annular channel with an inside diameter of 0.22 m, an outside diameter of 0.34 m, and a length of 0.5 m. Swirled air streams entered the working section from two coaxial chan-

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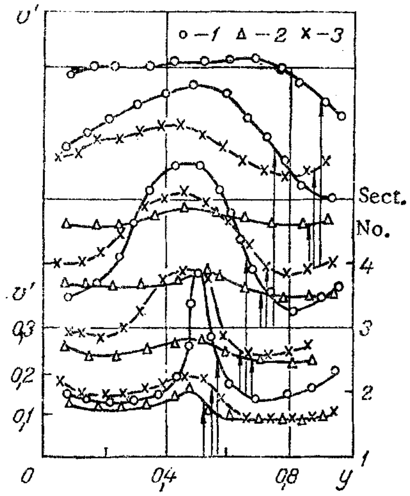

Fig. 1. Distribution of pulsation velocity over channel cross sections (N = 1, z = 0; 2, 43; 3, 100; 4, 200 mm): 1) $\varphi_m = 57°/\text{-}48°$; 2) 19.5°/–28°; 3) 41°/–37°.

nels separated by an annular shell 1.2 mm thick. The axial (v_z) and rotary (v_φ) components of the average velocity and the magnitude v' of the turbulent velocity pulsations in the direction of the average velocity vector were measured in the tests. The measurements were made with cylindrical three-point pneumometric probes and thermoanemometer probes. The experimental accuracy was 5-10%. The circular nonuniformity did not exceed 5%. The experimental installation and the procedure for making the tests are described in detail in [2].

In the investigated modes of flow the average-flow-rate velocity v_0 ranged from 25 to 30 m/sec, which corresponded to Reynolds numbers Re = v_0H/ν of from 10^4 to $1.2 \cdot 10^4$. In the tests the profiles of the axial components of the averaged velocity of each of the streams at the entrance to the working section were nearly uniform, while the degree of concurrent flow was $m = v_2/v_1 \approx 1$. (Here and later the indices 1 and 2 pertain to the inner and outer streams, respectively.) The profiles of the rotary components of the averaged velocity at the entrance had an approximately sinusoidal shape $v_\varphi = v_{\varphi_m} \sin(\pi y/H)$.

Graphs of the variation of the profiles of pulsation velocity v' over the channel length for different values of φ_m in the inner/outer streams, $v_{\varphi_m} = v_0 \tan \varphi_m$, are presented in Fig. 1. It is seen from the graphs that near the interface between the streams the pulsation velocity v' has a maximum, the value of which is the larger, the higher the relative velocity of motion of the two streams.

An analogous monotonic dependence of the maximum pulsation velocity v'_m on $\Delta v_{\varphi_m} = |\Delta v_{\varphi_{m_1}} - v_{\varphi_{m_2}}|$ also occurred for other angles of stream swirling. This is shown in Fig. 2, where the quantity v'_m is normalized to the average-flow-rate velocity v_0, while $\Delta S_m = \Delta v_{\varphi_m}/v_0$. It is seen from the figure than a good approximation of the dependence of v'_m on Δv_{φ_m} is the straight line

$$v'_m = 0.24 \Delta v_{\varphi_m}. \quad (1)$$

The departure of some experimental points from this straight line may be connected with the fact that equality of the degree of concurrent flow m to unity was not maintained quite exactly in the tests, as well as with overstatement of the thermoanemometer readings at high intensities of stream turbulence.

The relation (1), along with the relation

$$\frac{db}{dz} = 0.27 \frac{\Delta v_\varphi}{2v_0} \quad (2)$$

obtain in [2], establishes the connection between the magnitude of the turbulent velocity pulsations generated in the stream and the angle of expansion of the mixing layer. With uniform profiles of the rotary component of the averaged velocity in each of the streams, this connection has the form

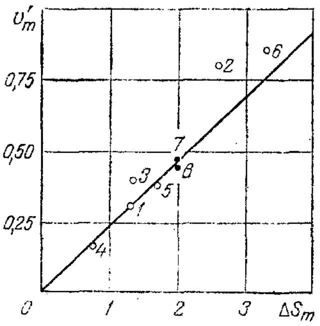

Fig. 2

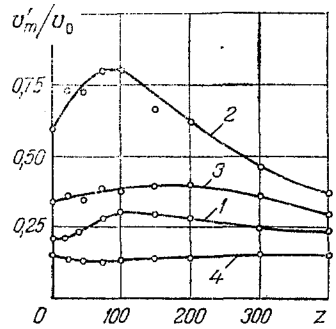

Fig. 3

Fig. 2. Dependence of the maximum value of the pulsation velocity on the relative swirling of the streams at the entrance: 1) $\varphi_m = 5°/\text{-}54°$; 2) 57°/–48°; 3) 54°/–5°; 4) 19.5°/–28°; 5) 41°/–37°; 6) 34°/–68°; 7) experiment [5]; 8) experiment [1].

Fig. 3. Variation of the intensity of turbulent pulsations along the channel length (points: experiment): 1) φ_m = 5°/–54°; 2) 57°/–48°; 3) 54°/–5°; 4) 16°/–21°. z, mm.

$$\frac{db}{dz} = 0.56\,\frac{v'_m}{v_0}\,. \tag{3}$$

It must be mentioned that this relationship holds only in the case when the level of the turbulent pulsations generated in the stream exceeds the turbulence intensity at the entrance to the channel. In addition, all the foregoing is valid for the mixing of streams swirled in the same direction if one can neglect the influence of rotation on the generation of turbulent pulsations in comparison with the influence of transverse shear in the profile of velocity v_φ.

Thus, in an analysis of the intensity of stream mixing we can use the quantity v'_m instead of the characteristic db/dz. We use this criterion to investigate the influence of the parameters of concurrence m and relative swirling ΔS on stream mixing when they act jointly. We define the swirling of a stream as follows: S = tan φ.

The relationships found in [2] for the mixing of swirled streams show that they are similar to the relationships displayed in ordinary shear flows. This is the basis for using the Prandtl mixing-length theory to analyze the mixing of swirled streams. In this case we must assume that the coefficient of turbulent exchange μ_t is proportional to the second invariant of the deformation-rate tensor of the averaged flow,

$$\mu_t = \rho L^2 \left[\left(\frac{\partial v_z}{\partial r}\right)^2 + \left(r\frac{\partial}{\partial r}\frac{v_\varphi}{r}\right)^2\right]^{0,5}, \tag{4}$$

where L is the mixing length.

It is clear that at low turbulence intensities at the entrance the maximum value of the turbulence energy will be determined by the size of the terms describing the generation of turbulent pulsations in the equation of transfer of turbulence energy k [4],

$$\operatorname{div}(\rho V k - \mu_t/\sigma_k \operatorname{grad} k) = \mu_t F_k - \rho\varepsilon, \tag{5}$$

where ε is the rate of dissipation of turbulence energy per unit volume; σ_k is the Prandtl–Schmidt number;

$$F_k = 2\left[\left(\frac{\partial v_z}{\partial z}\right)^2 + \left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{v_r}{r}\right)^2\right] + \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)^2 + \left(r\frac{\partial}{\partial r}\frac{v_\varphi}{r}\right)^2 + \left(\frac{\partial v_\varphi}{\partial z}\right)^2. \tag{6}$$

Thus, $(v'_m)^2 \sim k_m \sim (\mu_t F_k)_m$. Since for flows in channels the radial component of the averaged velocity is usually considerably smaller than its other components and $\partial/\partial z \ll \partial/\partial r$, we have $F_k \approx (\partial v_z/\partial r)^2 + (r\partial/\partial r \cdot v_\varphi/r)^2$. Consequently, the parameter determining the stream mixing is the sum

$$\Sigma = \left(\frac{\partial v_z}{\partial r}\right)^2 + \left(r\frac{\partial}{\partial r}\frac{v_\varphi}{r}\right)^2. \tag{7}$$

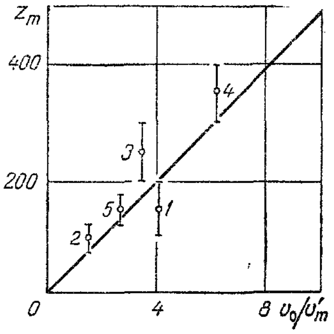

Fig. 4. Connection between the coordinate z_m (mm) and the quantity v'_m (points: experiment): 1) $\varphi_m = 5°/-54°$; 2) 57°/−48°; 3) 54°/−5°; 4) 41°/−37°; 5) 19.5°/−28°.

If we neglect the influence of stream rotation on the generation of turbulent pulsations in comparison with that of transverse shear, we obtain $\Sigma \approx \approx (\partial v_z/\partial r)^2 + (\partial v_\varphi/\partial r)^2$. Here the relative influence of the shear of the longitudinal and rotary components of the averaged velocity on the stream mixing will be characterized by the quantity $t = (\partial v_\varphi/\partial r)/(\partial v_z/\partial r)$. Because of the self-similarity of the velocity profiles in the mixing layer [2], $t = \Delta v_\varphi/\Delta v_z$, where Δv_φ and Δv_z are the differences between the components of the averaged velocity at the boundary of the mixing layer. Experiments [2] show that for the quantities Δv_φ and Δv_z one can take their maximum values in the stream:

$$\frac{\Delta v_z}{v_0} = 2\frac{|v_{z_{m_1}} - v_{z_{m_2}}|}{v_{z_{m_1}} + v_{z_{m_2}}}, \quad \frac{\Delta v_\varphi}{v_0} = \Delta S.$$

Then

$$t = \frac{\Delta S(1+m)}{2|1-m|}. \tag{8}$$

It is seen from Eq. (8) that for t = 1 the two parameters (the relative stream swirling ΔS and the degree of concurrence m) make the same contribution to the intensification of turbulent exchange between the streams, for t > 1 the swirling parameter ΔS makes the greater contribution, and for t < 1 the degree of concurrence m makes the greater contribution. We note that in terms of m and ΔS the stream mixing will be determined by the quantity

$$\Sigma = \Delta S^2 + 4\left(\frac{1-m}{1+m}\right)^2. \tag{9}$$

This criterion establishes the relation between the mixing intensities of swirled and concurrent streams. Thus, the mixing of unswirled streams with a degree of concurrence $m \neq 1$ will be characterized by the same angle of expansion of the mixing layer as the mixing of swirled streams with the same longitudinal velocity and a relative swirling.

$$\Delta S = 2\frac{|1-m|}{1+m}, \tag{10}$$

since the quantity Σ will be the same in both these cases.

Points calculated from the experimental data of [1, 5] for a mixing layer of flooded jets (m = 0) are plotted in Fig. 2. In treating the experimental data of [1] we assumed isotropy of the turbulent pulsations, i.e., $v'_x = v'_y$, $v' = (\langle v'^2_x\rangle + \langle v'^2_y\rangle)^{0.5}$. It is seen that the plotted points lie close to the straight line (1). Unfortunately, such a comparison could not be made for other values of the parameter m owing to the absence of the appropriate experimental data and because of the strong influence of boundary layers in the entrance cross section on the process of mixing of concurrent jets, especially for values of m close to unity. Another important parameter of the problem under consideration is the turbulence level at the entrance. According to the data of [1, 6], it influences stream mixing for 0.5 < m < 2.0, i.e., for 0 < ΔS < 0.7 in accordance with Eq. (10). The value of ΔS = 0.7

corresponds to a swirling angle $\varphi_m \approx 35°$. Thus, the influence of the entrance turbulence on the mixing of oppositely swirled streams should start to show up at swirling angles of about 30-35°, which is well confirmed by the present experiments.

In Fig. 3 we present experimental distributions of the maximum values of the pulsation velocity v' in a given cross section along the channel length for different modes of flow. It is seen from the graphs that for swirling at angles of 16°/−21° (curve 4) there is a decrease in the intensity of the turbulent pulsations in the section directly adjacent to the entrance. An increase in v'_m is observed for other swirlings (curves 1-3). These differences are explained by the fact that in the initial section there is, on the one hand, damping of turbulent pulsations, the presence of which at the entrance is connected with peculiarities of flow in swirling apparatus, and on the other hand, the generation of pulsations due to transverse shear in the profile of the rotary component of the averaged velocity. Generation dominates in the case of large swirlings (45° or more). We note that in the mixing of oppositely swirled streams, in contrast to the mixing of concurrent jets, the influence of the boundary layers is hardly felt. This is connected with the fact that in the given case the boundary layers do not create additional nonuniformity of the profile of the rotary velocity component, while the levels of turbulent pulsations in the stream core in the presence of swirling are usually comparable with or even higher than the pulsation level in the boundary layers, as the tests show.

It is seen from Fig. 3 that after reaching a maximum, the turbulent pulsations start to decrease, and the damping of the pulsations takes place the more intensely, the greater the relative swirling of the streams at the entrance. This distinguishes the mixing of oppositely swirled streams in channels from the mixing of free jets: Slow damping of turbulent pulsations along the length of the jet occurs in the latter case.

The distance from the entrance at which the turbulent pulsations reach the maximum value (we designate it as z_m) decreases monotonically as the relative swirling of the streams increases. If we approximate the boundaries of the mixing layer by straight lines $b \sim z \cdot \Delta v_\varphi / v_0$ and use the relation (1), then we find that $b_m/z_m \sim v'_m/v_0$ or, since $b_m \sim H$,

$$\frac{z_m}{H}\frac{v'_m}{v_0} = \text{const.} \tag{11}$$

Experimental dependences of the quantity z_m on the maximum value of the turbulent velocity pulsations v'_m in the stream are presented in Fig. 4. The approximation of these data by the straight line (11) yields a value of 0.8 for the constant. With allowance for (1), Eq. (11) can also be rewritten in the form

$$z_m = \frac{10}{3}\frac{H}{\Delta S_m}. \tag{12}$$

The relationships obtained in [2], together with the data presented in this article, allow one to estimate the values of the various turbulence characteristics. Thus, from Eqs. (2) and (3) and the fact that the profiles of the relative rotary component of the averaged velocity in the mixing layer are self-similar in length and are described by Schlichting profiles it follows that the value of the "mixing length" corresponding to v'_m is

$$l = 0.12\, b. \tag{13}$$

Since $b_m \approx 0.45H$ in the cross section $z = z_m$, we have $l_m = 0.054H$. We note that in obtaining the coefficient 0.12 in Eq. (13) we used the assumption that the turbulent pulsations are isotropic. If we assume that these pulsations are anisotropic and, just as in the mixing of unswirled concurrent jets, we have the relation $v'_r \approx (1.2\text{-}1.4)v'_\varphi$, then we obtain

$$l \approx 0.1\, b,\ l_m \approx 0.05\, H. \tag{14}$$

Consequently, at the place where the turbulence energy has the maximum value in the stream, the width of the mixing layer equals the half-width of the channel, while the "mixing length" is $l_m \approx 0.05H$. A comparison with the experimental data of [2] shows that this value is the maximum for the given flow. Thus, the confinement of the flow plays a significant role in the mixing of streams in pipes and channels, in contrast to the mixing of free jets.

For the turbulent viscosity ν_t we have the following estimates: $\nu_t = 0.09k^{0.5}l \approx 2\cdot 10^{-3}\Delta v_\varphi b$ or $\nu_{Tm} = 10^{-3}\Delta v_\varphi H$.

Thus, there is also an analogy in the turbulence characteristics between the mixing of oppositely swirled and concurrent streams.

NOTATION

z, r, φ, cylindrical coordinate system; v_z, v_r, v_φ, components of the averaged velocity; v', pulsation component of the velocity in the direction of the averaged velocity vector; v_0, average-flow-rate velocity of flow; H, radial gap of the annular channel; y, radial distance from the inner wall; b, width of mixing layer; m, degree of concurrent flow; S, stream swirling; μ, ν, coefficients of viscosity. Indices: 0, average-flow-rate; 1, inner; 2, outer stream; m, maximum; t, turbulent.

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CALCULATION OF THE CHARACTERISTICS OF A MIXING CO_2 GASDYNAMIC LASER WITH A NOZZLE UNIT OF HONEYCOMB CONSTRUCTION

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A mathematical model of a gasdynamic laser with parallel supersonic mixing of the components and its applications to the choice of the geometrical characteristics of the nozzle unit of honeycomb construction are presented.

One of the main problems in the creation of gasdynamic lasers (GDL) based on mixing is working out the construction of the nozzle apparatus simultaneously assuring the fullest realization of the energy stored in the exciting gas (N_2, CO), good optical quality of the medium, and sufficient pressure for exhausting into the atmosphere. From this point of view, the use of nozzle units of honeycomb construction, consisting of a large number of small-scale, axisymmetric nozzles having separate supply of the exciting and radiating gases with parallel supersonic mixing of the components has recently evoked great interest [1]. However, the available experimental results were obtained on specific constructions and evidently do not reflect the potenialities of this scheme. On the other hand, a unit of honeycomb construction, thanks to its exceptional simplicity and technological effectiveness, can be built with the most varied geometrical characteristics.

The composite gasdynamic pattern of formation of the supersonic stream of active medium, the interaction of the supersonic axisymmetric jets of the individual nozzles and the presence of wakes behind their rims, and the finite size of the mixing zones can have different and very significant influences on the characteristics of GDL. The use of the combustion products of various fuels, usually containing a number of admixtures besides nitrogen promoting the deactivation of the vibrationally excited molecules even before the interaction with the radiating gas, as the exciting gas requires great caution when using the assumption of "instantaneous mixing" to determine the GDL characteristics, especially from the aspect of their optimization. The model of "instantaneous mixing" was used in [2, 3] with certain additional assumptions for high values of the ratio of flow rates of the radiating and donor gases in [4]

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